EXPERIMENTAL STUDY OF CONCRETE AGING EFFECT ON THE CONTACT FORCE AND CONTACT TIME DURING THE IMPACT INTERACTION OF AN ELASTIC ROD WITH A VISCOELASTIC BEAM

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ABSTRACT

In the present paper, the low-velocity impact of an elastic rod with a flat end upon a viscoelastic Timoshenko type beam has been considered. Viscoelastic properties of the beam out of the contact zone are described by the standard linear solid model with integer derivatives, while inside this zone they are governed by the fractional derivative standard linear solid model. The contact force for a concrete target has been defined experimentally at the concrete age of 7, 14, 28, 56, and 91 days. It has been found that an average maximum of the contact force increases with concrete age, whereas the contact duration decreases. Moreover, the most remarkable changes of both, contact force and contact time, occur at the concrete age earlier than 14 days, after that the rate of changes slows down. Experimental results have a good coincidence with theoretical expectations.

Keywords: Contact force, Contact time, Impact interaction, Viscoelastic model.

1. INTRODUCTION

The problems associated with the impact interaction of thin bodies (rods, beams, plates, shells) with other bodies, are widely used in various fields of science and technology. Since these tasks are related to the problems of dynamic contact interaction, the solution involves considerable mathematical and computational difficulties. In order to overcome those difficulties many different methods reviewed in [1] and [2] have been proposed.

In the last three decades, the use of fractional calculus has attracted considerable interest in various fields of natural sciences and technology. The state-of-the-art article [3] presents a thorough analysis of the application of fractional calculus to the dynamic problems of linear and non-linear solids and structures, including the problems of shock interaction.

In the present paper, viscoelastic properties of a Timoshenko isotropic beam outside the area of contact interaction are described by the standard linear solid model with conventional derivatives of integer order. Viscoelastic properties inside the contact area are governed by the standard linear solid model with fractional derivatives. A fact that decrosslinking could occur between molecules of beam's material within the contact area during the impact leads to more free motion of molecules in relation to each other and consequently

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reduces the viscosity in the contact area. The order of the fractional derivative is the additional constructive parameter, which allows one to control the viscosity of beam's material [4]. Since concrete properties continuously change with time, it is also important to know how aging of material affects the impact process, namely, the contact force and contact duration. In order to answer this question, a set of experiments has been conducted.

The problem, which is solved in this paper, is rather unique. The matter is fact that in the majority of papers (see for example, [5,6]) dealing with experimental determination of contact characteristics during the impact interaction viscoelastic properties of the materials of interacting bodies are not taken into account. From the other hand, studies presenting measurements of viscoelastic material properties [7, 8], including the effect of materials' aging [9, 10], do not refer to the impact interaction problems. To the authors' knowledge, there are no papers devoted to experimental measurement of the contact force and contact duration for concrete material and their changes with age, taking into account viscoelastic properties of the material via fractional calculus model.

2. THEORETICAL BACKGROUND

2.1 Problem Formulation

Let a long prismatic elastic rod of rectangular cross-section with dimensions $2\tau_{im}$ and a move along the y -axis with velocity V_0 toward an isotropic rectangular beam of infinite length (Fig. 1). The last assumption takes place due to the short duration of the impact interaction and allows one to neglect reflected waves.

Impact occurs at the moment $t = 0$ at the origin of the coordinate system x, y, z. At the moment of impact, transient waves are initiated in the beam and rod, which later propagate along the impactor and the target with the velocities of transient waves.

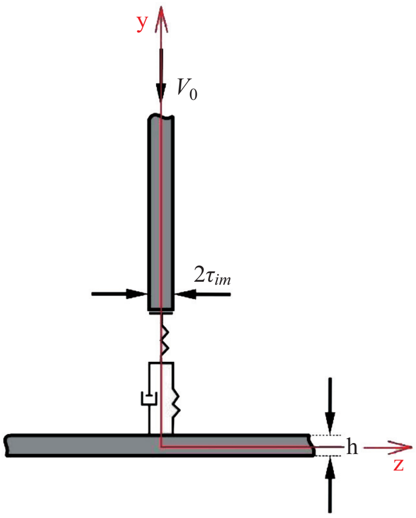

Fig. 1 Scheme of the viscoelastic interaction.

Dynamic behavior of a viscoelastic Timoshenko-type beam with consideration for rotary inertia and transverse shear deformations could be described by the following system of equations [4]:

$$\frac{\partial Q}{\partial z} = \rho A \dot{W}, \quad Q = K\mu_\infty A\left[\varphi - \frac{\nu_\mu^\varepsilon}{\tau_\mu^\varepsilon}\int_0^t e^{-\frac{t-t'}{\tau_\mu^\varepsilon}}\varphi(t')dt'\right], \tag{1}$$

$$\frac{\partial M}{\partial z} - Q = \rho I \dot{\beta}, \quad M = -E_\infty I\left[\frac{\partial \beta}{\partial z} - \frac{\nu_E^\varepsilon}{\tau_E^\varepsilon}\int_0^t e^{-\frac{t-t'}{\tau_E^\varepsilon}}\frac{\partial \beta(t')}{\partial z}dt'\right], \tag{2}$$

where M is the bending moment, Q is the lateral force, $W = \dot{w}$ is the velocity of deflection of the points at the middle plane, β is the rotation angle around the z-axis of the cross-section, $\varphi = \partial w/\partial z - \beta$, ν_E^ε, ν_μ^ε, τ_E^ε, and τ_μ^ε are material constants, E_∞ and μ_∞ are non-relaxed values of elastic and shear moduli, respectively, ρ is beam's material density, K is the shear coefficient, A and I are cross-section area and moment of inertia, respectively, and an overdot denotes time derivative.

It is necessary to add equations describing the dynamic behavior of the impactor to equations (1) and (2):

$$\frac{\partial \sigma}{\partial z} = \rho_{im}\dot{v}, \quad \dot{\sigma} = E_{im}\frac{\partial v}{\partial z}, \tag{3}$$

where σ is the stress, v is the velocity, ρ_{im} and E_{im} are impactor's material density and Young's modulus, respectively. The equation of motion of the contact area with the length of $2\tau_{im}$, as well as the equation for the contact force F_{cont} [11] must be added (Fig. 1):

$$2\tau_{im}A\rho\ddot{w} = 2Q|_{z=\tau_{im}} + F_{cont}, \tag{4}$$

$$F_{cont} = E_\infty(\alpha - w) - \Delta E t\int_0^t \ni_\gamma\left(-\frac{t-t'}{\tau_\varepsilon}\right)\left[\alpha(t') - w(t')\right]dt', \tag{5}$$

where α and w are displacements of upper and lower ends of the spring, respectively, the displacement w equals to the beam displacement at the contact point, τ_ε is the relaxation time, $\Delta E = E_\infty - E_0$ is the defect of the elastic modulus, E_0 is the relaxed Young's modulus, γ ($0 < \gamma \leq 1$) is the fractional parameter,

$$\ni_\gamma\left(-\frac{t}{\tau_\varepsilon}\right) = \frac{t^{\gamma-1}}{\tau_\varepsilon^\gamma}\sum_{n=0}^\infty \frac{(-1)^n (t/\tau_\varepsilon)^{\gamma n}}{\Gamma[\gamma(n+1)]} \tag{6}$$

is Rabotnov's fractional exponential function [12], and $\Gamma(\gamma)$ is the gamma-function. Equation (6) is equivalent to the standard linear solid model with fractional derivatives [3].

There is a need to add initial conditions to the system of equations (1)-(5):

$$\alpha|_{t=0} = w|_{t=0} = \dot{w}|_{t=0} = 0, \quad \dot{\alpha}|_{t=0} = V_0. \tag{7}$$

2.2 Solution Method

For the solution of this problem two methods could be used: the ray method and Laplace transformation. The ray method is applicable for the construction of an approximate solution for the beam behind the shock wave front till the boundary of the contact area, as well as for construction of the exact solution in excited area of the elastic rod. Within the contact area, the contact force could be defined by the Laplace transformation.

In order to find the solution out of the contact zone in both, the beam and the rod, the compatibility condition [13]

$$\dot{Z} = -G\frac{\partial Z}{\partial z} + \frac{\delta Z}{\delta t}, \tag{8}$$

and the polynomial ray expansions for the unknown functions Z are employed, where G is the normal velocity of the wave surface and $\delta Z/\delta t$ is the δ-time derivative [2, 13]. As a result, the following expressions for the lateral force on the boundary of the contact zone and the contact stress at the end of the rod are obtained in the form

$$Q = -\rho A G_\infty W, \tag{9}$$

$$\sigma_{cont} = \rho_{im} G_{im}\left(V_0 - W - \dot{\alpha}\right), \tag{10}$$

where $G_\infty = \sqrt{K\mu_\infty / \rho}$ and $G_{im} = \sqrt{E_{im} / \rho_{im}}$ are propagation velocities of the quasi-transverse shock wave in the beam and the longitudinal shock wave in the rod, respectively. When the contact stress (10) is known, the contact force can be found as:

$$F_{cont} = b\left(V_0 - W - \dot{\alpha}\right), \tag{11}$$

where $b = 2a\,\tau_{im}\,\rho_{im}\,G_{im}$.

Equations (5), (11) and (4) rewritten according to (9) in the form

$$M\ddot{w} + MB\dot{w} = F_{cont}, \tag{12}$$

where $B = G_\infty/\tau_{im}$ and $M = 2\tau_{im}A\rho$ is the mass of the contact area, represent a closed system of equations in three unknowns: F_{cont}, w and α.

Applying the Laplace transformation to the set of equations (5), (11) and (12), going reverse from the Laplace domain to the time domain, using the theory of residues within the first sheet of the Riemann surface, the contact force can be found as

$$F_{cont}\left(t\right) = \int_0^\infty \frac{1}{s}B(s)(1 - e^{-st})ds + \frac{A}{\sqrt{\kappa^2 + \omega^2}}[-\sin(\phi + \phi_0) + e^{-\kappa t}\sin(\omega t + \phi - \phi_0)], \tag{13}$$

where $-\kappa \pm i\omega$ are simple poles at the first sheet of the Riemannian surface, $\tan\phi_0 = -\omega/\kappa$, and functions $B(s)$, A and ϕ are not shown due to their awkwardness. The details of this problem are given in authors' recent paper [4].

The first term in the expression (13) defines the drift of equilibrium position, and the second one governs damped vibrations around the drifting equilibrium position.

The dimensionless time $t^* = \omega t$ dependence of the dimensionless contact force $F^*_{cont} = F_{cont}\sqrt{\kappa^2 + \omega^2} / A$ [4] is shown in Fig. 2 for different values of the fractional parameter, the values of which are shown by digits near the corresponding curves. Values $\gamma = 0$ and $\gamma = 1$ correspond, respectively, to an elastic and viscoelastic Timoshenko beam, the properties of which can be described by the standard linear solid model with regular viscosity.

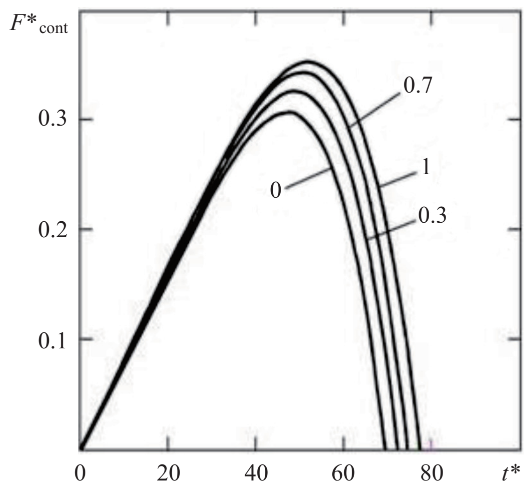

Fig. 2 The dimensionless time dependence of the dimensionless contact force [4].

3. EXPERIMENTAL PROGRAM

In order to determine the effect of aging on the contact force and contact time during the impact interaction between an elastic rod and a viscoelastic beam, three concrete beams (one meter in length and squared cross-section 100 mm × 100 mm) were cast. Mix proportions per cubic meter, which are designed according to the ACI standard C211.1 with the water to cement ratio of 0.35 and expected compressive strength at 28 days of 47 MPa, were given as follows: water – 186.88 kg; cement –533.95 kg; fine aggregates – 608.80 kg (SSD); coarse aggregates – 1055.57 kg (SSD) and 2.20 kg of Type G superplasticizer. Slump of fresh concrete mixture reaches 145 mm (see Fig. 3). At the same time, a batch of cylindrical concrete samples (200 mm in height and 100 mm in diameter) was cast for the standard non-destructive tests and compressive strength tests as well. After 24 hours, samples were demolded and stored in the water. At the age of 7 days samples were extracted from the water, and beams were fixed on the experimental position as shown on Fig. 4.

Fig. 3 Beams casting and fresh concrete performance.

Fig. 4 Experimental setup.

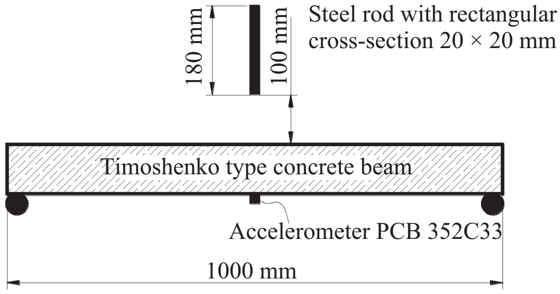

Fig. 5 Impact test scheme.

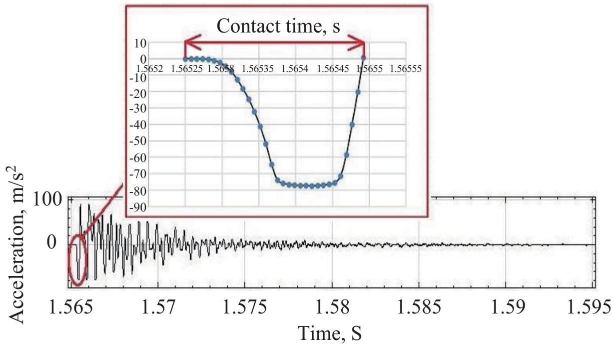

Fig. 6 Graphical measurement of the contact time from the acceleration vs. time diagram.

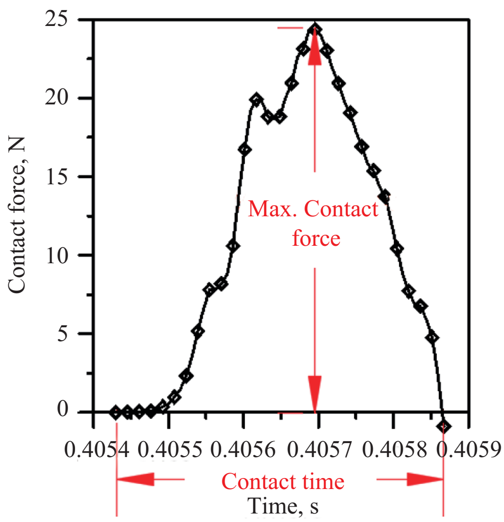

Fig. 7 Graphical identification of the contact force during the impact process.

The main purpose of the test was to identify the contact force and contact time during the impact interaction between an elastic steel rod and a viscoelastic Timoshenko-type concrete beam, as well as to define how these parameters were affected by aging. The detailed experimental scheme was shown in Fig. 5. An elastic rectangular steel rod (with cross-section dimensions of 19.7 × 19.42 mm, 181.34 mm in length and with mass m = 0.5197 kg) fell down from the height of 100 mm on a simply supported Timoshenko-type concrete beam. Normally, a simply supported beam, when impacted by a relatively heavy impactor, jumped up, and the signal could be destroyed. In order to prevent this effect, the beam was tightly bound to the support at the ends (see Fig. 4). Such a fixing method allowed the rotation around supports to prevent vertical displacements of the beam.

Accelerometer PCB 352C33 was attached to the middle of the bottom surface of the beam, right under the impact point and was measuring the acceleration of the beam during all the time when the beam was vibrating. The signal obtained from the sensor entered into data acquisition system AD-Link, and after transformation it could be visualized on a PC using Visual Signal software. The software had a function of export signal data into MS Excel file.

After MS Excel data file was ready, a graphical identification of the contact time could be done. It was measured graphically as a difference between corresponding data points [14] as shown in Fig. 6. Since the acceleration of the beam is known, the contact force could be calculated by using the Newton's second law (see Fig. 7).

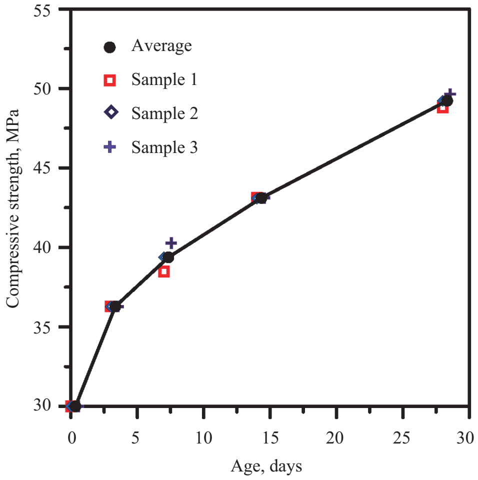

Fig. 8 Compressive strength development.

All samples were tested at the following ages of concrete: 7, 14, 28, 56 and 91 days. When conducting impact tests with concrete beams at very early age, it was important to ensure that the beam would not break down under the load not only due its self-weight, but also under the impact excitation. In order to verify that concrete beam surface would not have any obvious damage, a set of preliminary experiments was conducted. Compressive strength, as a good criterion of the estimation whether the sample was ready for the impact test or not, was implemented. Compressive strength of cylindrical concrete samples at the age of 7 days revealed the magnitude of 39.38 MPa (which was strong enough to start the impact test), and this value reached the magnitude of 49.22 MPa at the age of 28 days. Compressive strength development is shown in Fig. 8.

4. RESULTS AND DISCUSSIONS

Summarizing the experimental data obtained at the age of 7, 14, 28, 56 and 91 days (3 samples have been tested at each age), the time-dependence of the average contact force and average contact duration are plotted in Fig. 9. It is seen that these two parameters have two opposite trends: the contact force increases with concrete age, whereas the contact time decreases. At the age of 14 days, contact force is increased by 21.9% comparing to the value obtained at the age of 7 days, and it keeps increasing up to 27.5% at the age of 91 days. Opposite, at the age of two weeks contact time has decreased by 37.5%, and this value attains 45.1% at 91 days. Standard deviation varies from 0.101 to 0.257 for contact force test data, and from 0.344 to 2.314 for contact time, depending on the age. The concrete compressive strength dependence of average values of the maximal contact force and contact time is shown in Fig. 10, from which it is obvious that both of them have the same trends as in Fig. 9. The former increases and the latter decreases with the increase in the compressive strength. Since each single value of the compressive strength is taken at the corresponding age, the vertical scale of Fig. 10 is exactly identical to that of Fig. 9. The horizontal scale has been changed by the replacement of the age-axis by the compressive strength-axis. This is an additional way to verify the correctness of the obtained results. From both figures, these two trends can be explained by the effects of gradual hardening of concrete with the increase in ages. Fresh concrete as a viscoelastic material has dominating viscous properties, while the effect of elastic properties is minor. At this stage, the average contact force is the lowest and the average contact duration is the longest. After two weeks of concrete hardening, the material gains more elastic features, reducing its viscous properties. It is obvious that the most significant changes of both, contact time and contact force, occur at very early age, before 14 days.

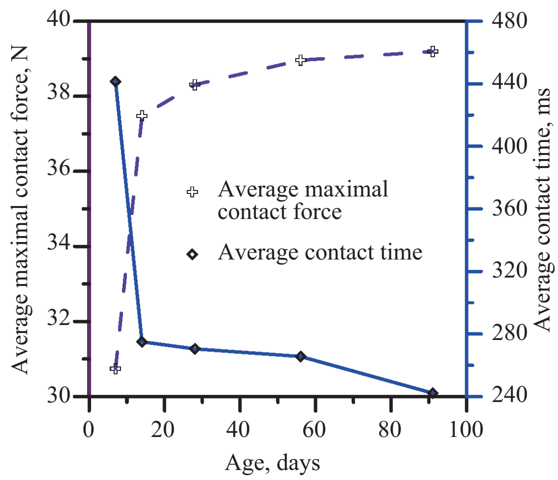

Fig. 9 Average contact time and average maximal contact force vs. time.

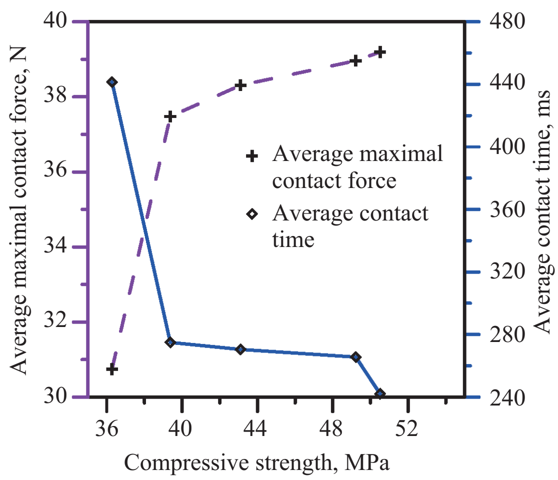

Fig. 10 Dependence of average contact time and average maximal contact force on the compressive strength of concrete.

Scientific value of the obtained results is of current interest from the view point of dynamic problems of impact interaction with due account for material's viscoelasticity. The results could be applied in calculation of local contact stresses and in prediction of possible material microstructural damage due to the accidental impacts on structural elements during construction and their service life.

5. CONCLUSIONS

Authors in the recent paper [4] have determined the dimensionless time dependence of the dimensionless contact force using analytical approach. In the present paper, such a dependence was obtained experimentally in real units (Fig. 7). The analysis of the experimental data at different age has revealed that the average maximal contact force increases with aging, whereas contact time decreases. The most remarkable changes occur at early age, before 14 days. After 2 weeks of concrete hardening, both the contact duration and contact force change slow down. Such a situation was expected due to the concrete hardening. When the material is soft, it behaves more like a viscoelastic material. After a certain period of hardening, the viscous behavior is reduced, and material behaves more like an elastic material. Theoretical model proposed in [4] involves the fractional parameter, which decreases with concrete age, and therefore it could be used as a criterion to describe the variations of material's viscosity due to its aging. Such a model has a good agreement with the experimental results and could be applied in engineering practice.

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